Local level statistics

Free probability meets supersymmetry

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Abstract

Our goal is description of local eigenvalue statistics of invariant random matrix models. We consider $N \quad N$ random matrices, governed by a confining analytic potential $V(H)$, and study the characteristic function in the $N \; ! \; 1$ limit. We use supersymmetry method to obtain behavior of many-point correlation functions. An important lemma, existence and uniqueness of supersymmetric Laplace Transform, has been proven for the functions in question. This technique is a powerful method of determining universality classes in correlations of eigenvalues.

Introduction

Invariant random matrix ensembles:

$$d_N(H) \; / \; e^{N\mathrm{Tr}V(H)}dH: \tag{1}$$

Characteristic function:

$$(K) = \int e^{\mathrm{Tr}KH}d\ (H): \tag{2}$$

For analytic and convex V we have saddle point equation [1]:

$$Q^{1} + R(Q) = z\mathrm{Id}_{pjq} \tag{3}$$

where Q is a rank (pjq) supermatrix argument of lifted $\hat{}(Q)$ required by supersymmetry method and R is a free probabilistic R-transform. Correlation functions may be retrieved from $\hat{}$ by analog of Laplace Transform:

$$\int \frac{\prod_{b=1}^{q}\mathrm{Det}(w_{1;b} \quad H)}{\prod_{a=1}^{p}\mathrm{Det}(w_{0;a} \quad H)}d_N(H) \; / \; \int \mathrm{SDet}^N(Q)\hat{}(Q)\,e^{\mathrm{STr}wQ}DQ \tag{4}$$

Objectives

1. Establish existence and uniqueness of Laplace Transform
2. Analyze singularities of R-transform
3. Obtain minimal requirements for the formalism
4.