A Riemann-Hilbert approach to the Muttalib-Borodin ensemble

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The model

The Muttalib-Borodin ensemble [2] with parameter > 0 and weight function w is the following probability density function:

$$\frac{1}{Z_n}\prod_{j<k}(x_k \quad x_j)(x_k \quad x_j)\prod_{j=1} w(x_j); \quad x_j \quad 0:$$

We consider an n-dependent weight function

$$w(x) = x \; e^{\;\; nV(x)}$$

with > 1 and an external eld V. The ensemble is a **determinantal point process**, it can be written as

$$\det \; K_{V;n}(x_i; x_j) \quad {}_{1 \;\; i;j \;\; n}$$

where $K_{V;n}(x;y)$ is the so-called correlation kernel.

Known result and main interest

Our main interest is to study the large n behavior of $K_{V;n}(x;y)$. Borodin [2] computed the **hard edge scaling limit** for the Laguerre case, namely if $V(x) = x$, then

$$\lim_{n ! 1} \frac{1}{n^{1+1=}} K_{V;n} \quad \frac{x}{n^{1+1=}}; \frac{y}{n^{1+1=}} \quad = \mathbb{K}^{(;)}(x;y)$$

with limiting correlation kernel

$$\mathbb{K}^{(;)}(x;y) = \quad y \quad \int_0^1 J_{\frac{+1}{},\frac{1}{}}(ux) J_{+1;}(uy) \quad u \; du$$

where

$$J_{a;b}(x) = \sum_{j=0} \frac{(\;\; x)^j}{j! \;\; (a+bj)}:$$

The same limit turns up in products of random matrices [1,4,5]. From these models and others [7] we know that the limit can be expressed in terms of **Meijer G-functions**.

By **universality** the limit is expected to hold for a much larger class of external elds V and our goal is to prove this.

[1]