

One-parameter deformation of Wishart – Laguerre ensemble

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Motivations

- Empirical or experimental data (e.g. in finance or QCD) can be modelled by positive definite random matrices (covariance or chiral models).
- Often, their spectral properties follow a power law distribution (time series of assets – instantons and delocalization in QCD etc...).
- Few solvable model so far for positive definite random matrices with fat tails!
- A deformed Wishart-Laguerre ensemble (**exactly solvable!**) can serve the purpose....

Power Laws and Random Matrices

- P

Wishart – Laguerre ensemble

$$P(\lambda_1, \dots, \lambda_N) \propto \prod_{i=1}^N e^{-a\lambda_i} \lambda_i^\nu \prod_{j < k} |\lambda_j - \lambda_k|^2$$

One-parameter deformation

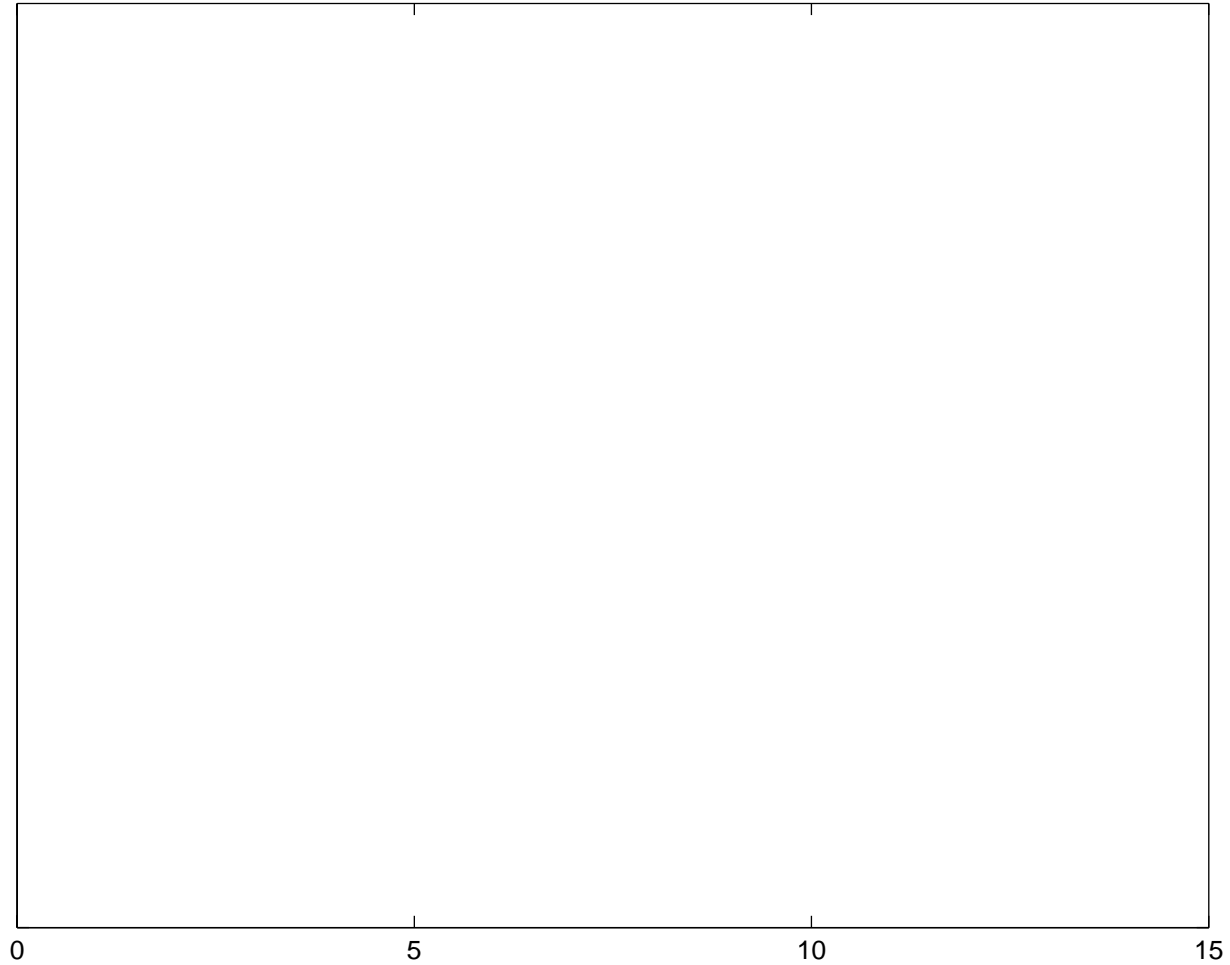
$$P_{\gamma; (N, \nu)}(\lambda_1, \dots, \lambda_N) \propto \left(1 + \frac{a}{\gamma} \sum_{i=1}^N \lambda_i\right)^{-\gamma} \prod_{i=1}^N \lambda_i^\nu \prod_{j < k} |\lambda_j - \lambda_k|^2$$

$\gamma \rightarrow \infty$ Back to Wishart - Laguerre

Gamma-transform Representation

$$1 + \frac{a}{\gamma} \text{Tr } X^{-\gamma} = \frac{1}{\Gamma(\gamma)} \int_0^{\infty} d\xi e^{-\xi} \xi^{\gamma-1} \exp \left[-\frac{a\xi}{\gamma} \text{Tr } X \right]$$

Smallest Eigenvalue (Finite N)



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positive integer for Wishart-Laguerre Unitary Ensemble: results by Dumitriu in terms of hypergeometric functions of a matrix argument.

For $\nu=0$, simpler expression available

$$\text{Prob}[x < \lambda_{\min, N}^{(\nu=0)} < x + dx] = aN \exp(-aNx)$$

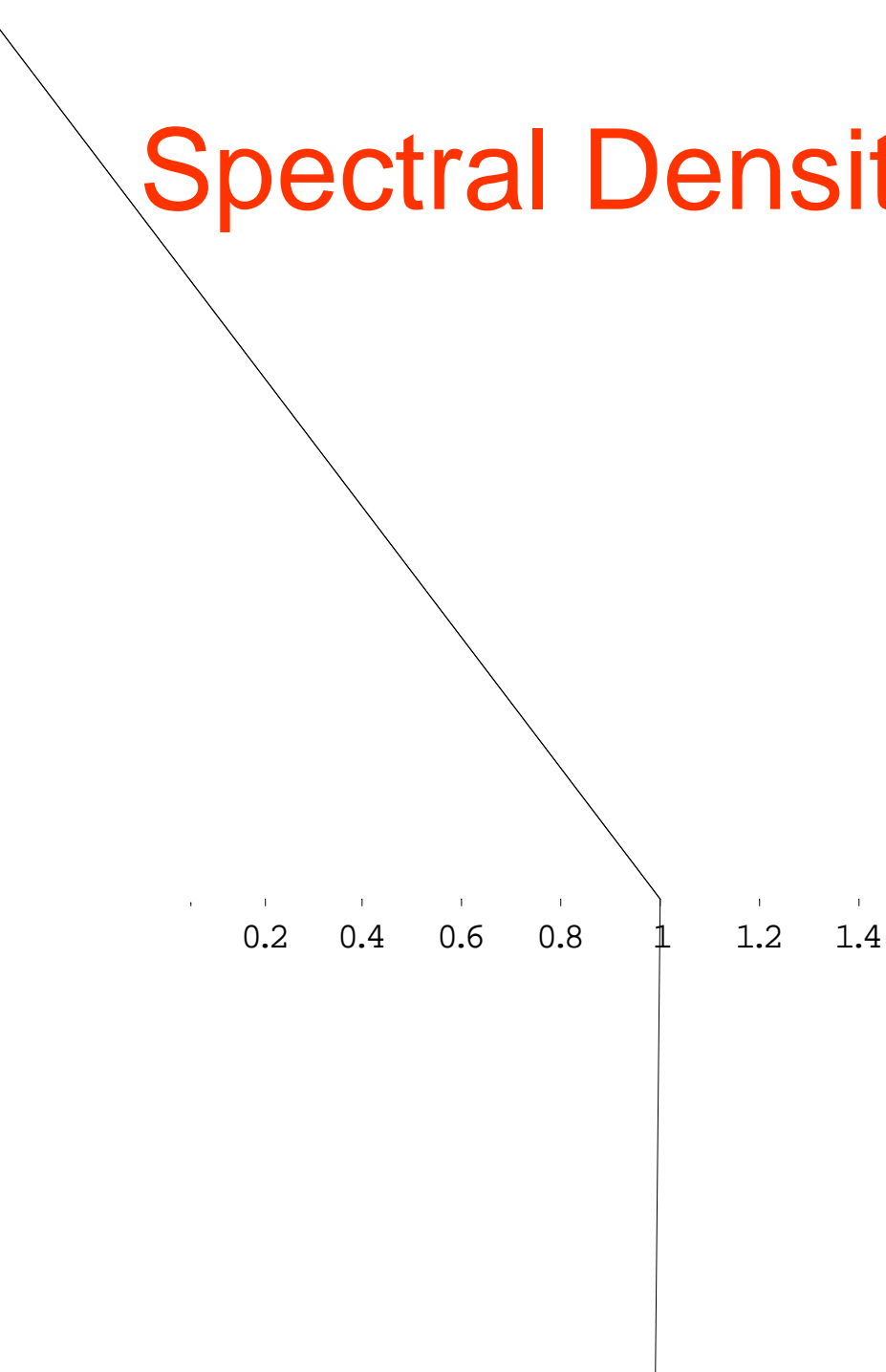
which gets modified in the deformed case as:

$$\text{Prob}[x < \lambda_{\min, N, \gamma}^{(\nu=0)} < x + dx] = \frac{aN(\gamma-1)}{\gamma} \left(1 + \frac{aN}{\gamma} x\right)^{-\gamma}$$

Spectral Density (Macroscopic)

- Modification of the large N limit due to the constraint
- One can take

Spectral Density (Macroscopic)



0.2 0.4 0.6 0.8 1 1.2 1.4

References

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