* + * ,$ -) M. / .A * SP - 0 /) A
and
A * D -) S+ * L+0AL.1A/.+ *

Luca G Molinari
Physics Department
Universita' degli Studi di Milano

Abstract: a determinantal identity (spectral duality and !ensen's theorem imply a "ormula "or the e#ponents o" a single generic trans"er matri# in terms o" the spectrum o" the corresponding $amiltonian matri#% &ith non $ermitian boundary conditions' Applications to Anderson model and ( ) M are presented'

Brunel, 20 dec 08

summary

- " eterminant# $% &l$c' tridia($nal matrice# and #pectral dualit)
- *i%t))ear# $% +nder#$n L$cali,ati$n
- -en#en.# the$rem and the #pectrum $% (L)apun$v! e/p$nent#
- 0ner() #pectra $% n$n 1ermitian +nder#$n matrice# and B2M
- the #malle#t e/p$nent

(LGM, arXiv:0808.1241 [math-ph !

determinants
o" tridiagonal matrices

$$\begin{bmatrix} a_1 & b_1 & & \\ c_1 & \ddots & \ddots & \\ & & & \\ & & & a_n \end{bmatrix}$$

$$\det \begin{bmatrix} a_1 & b_1 & & c_0 \\ c_1 & \ddots & \ddots & \\ & & & \end{bmatrix} = \cdots (-1)^{n+1} \cdots$$

determinants

o" bloc2 tridiagonal matrices

[redacted]

$$\det \mathrm{M}(z) = \frac{(-1)^{nm}}{[illegible]} \det[\mathrm{T} - zI_{2m}] \det[B_1 \cdots B_n]$$

$$\det \mathrm{M}^{(0)} = (-1)^{nm} \det[\mathrm{T}_{11}^{(0)}] \det[B_1 \cdots B_{n-1}]$$

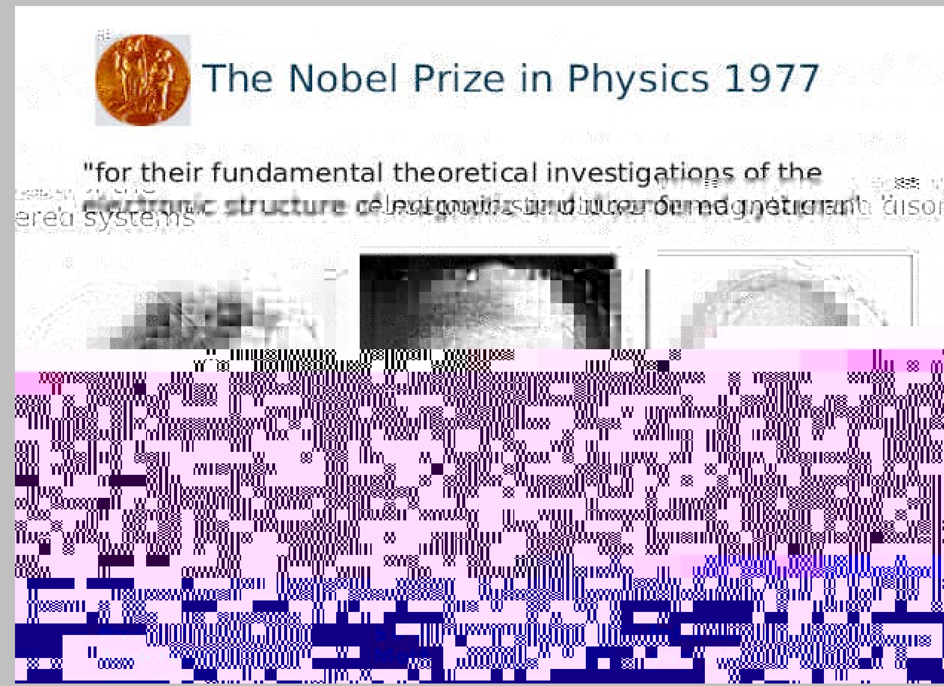
$$\mathrm{T} = \begin{bmatrix} -B_n^{-1}A_n & -B_n^{-1}C_{n-1} \\ I_m & 0 \end{bmatrix} \cdots \begin{bmatrix} -B_1^{-1}A_1 & -B_1^{-1}C_0 \\ I_m & 0 \end{bmatrix}$$

Theorem 1 (The Duality Relation)

$$\det[\lambda I_{nm} - \mathrm{M}(z)] = (-z)^{-m} \det[\mathrm{T}(\lambda) - zI_{2m}] \det[B_1 \cdots B_n]$$

L.G.M, Linear +l(e&ra and it# +pplicati$n# 423 (2008! 2221

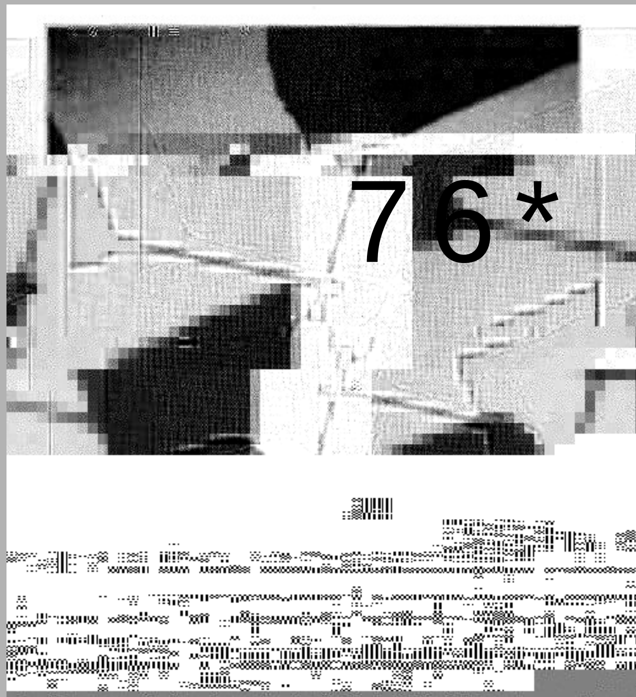

ices
Absence of Diffusion in Certain Random Latt

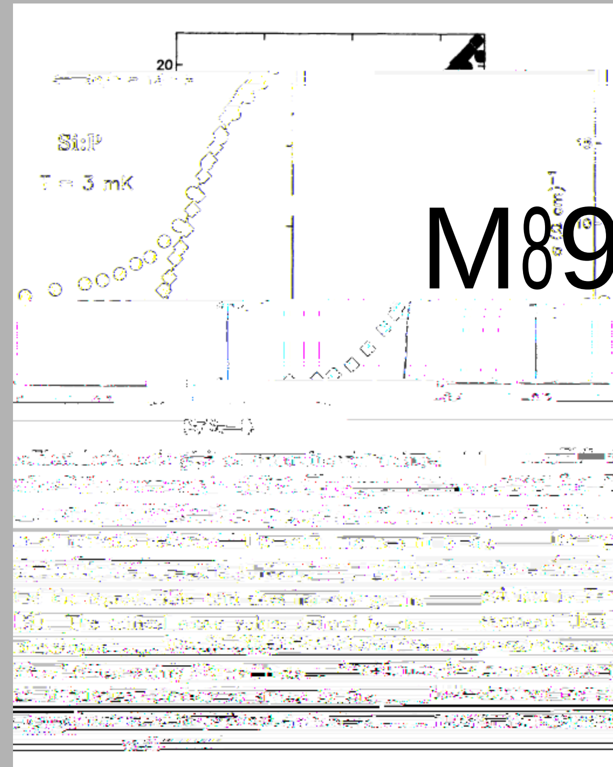


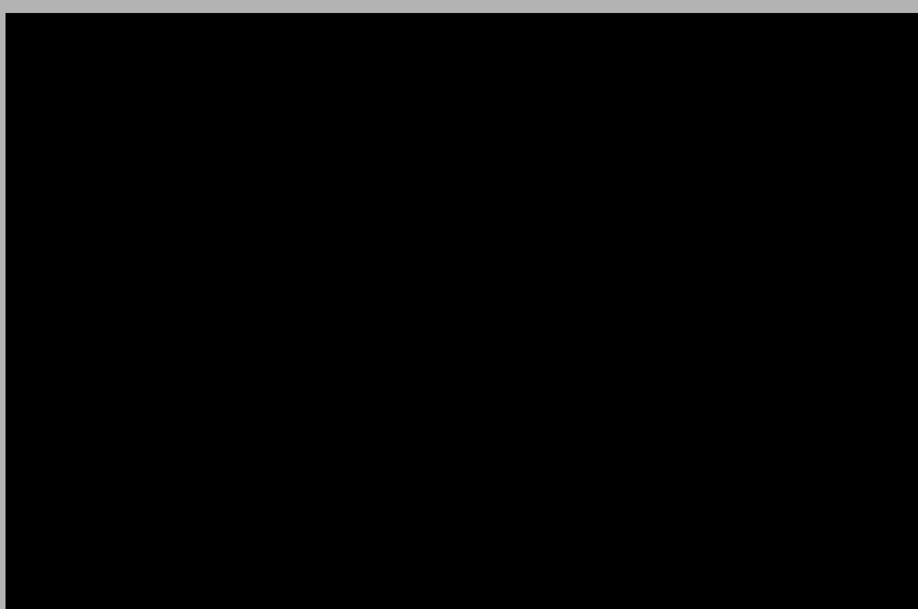
.saac *e&ton .nstitute "or Mathematical Sciences

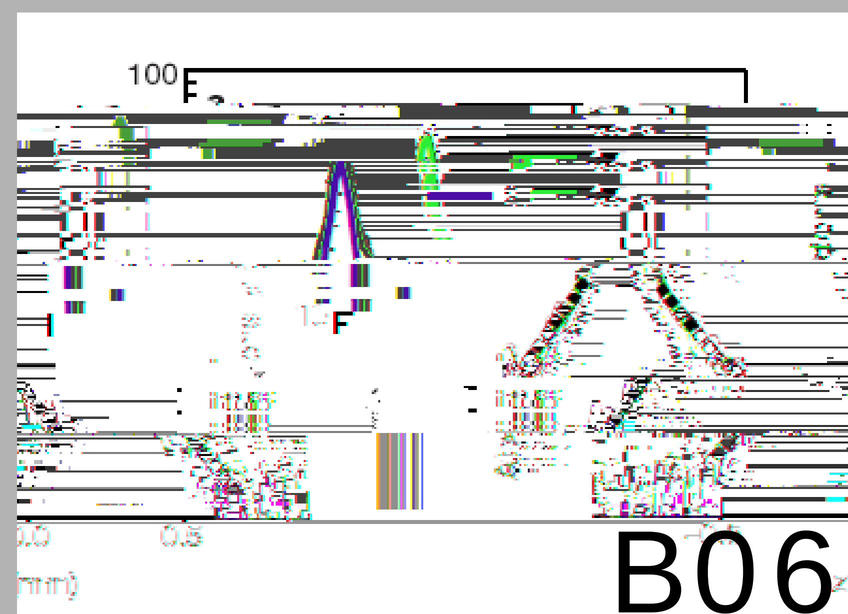
Mathematics and Physics o" Anderson locali3ation: 45 6ears A"ter

14 -ul) - 13 "ecem&er 2008







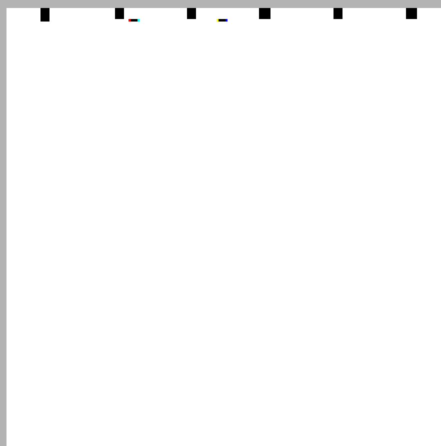


- 7UA*/UM 0$A+S: d)namical l$cali,ati$n
- #$und-li(ht-matter 4ave#

the$r):

- /heorems (:pen
- 8ubo "ormula in
+lt#huler, ...!
- -nergy levels and b
1atan$; <el#$n m$d., cu
- /rans"er matri# and Lya
#calin((=ramer ; Mac=inn$n!,
c$nductan n

A * D -) S + * M + D - L:



$$\sum u \cdot + v_n u_n = E u_n$$

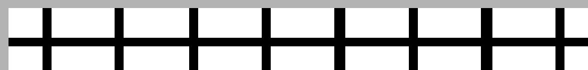
[redacted]

[redacted]

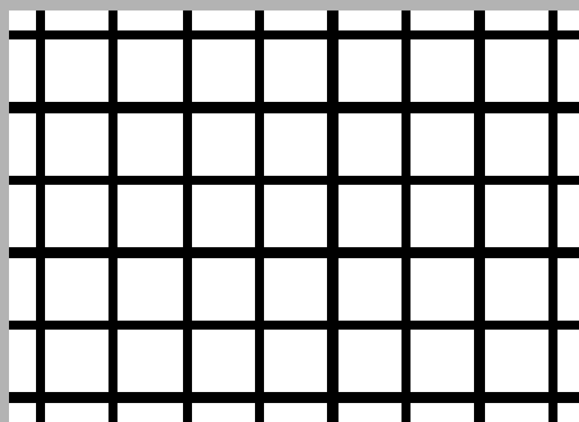
$$|u_n| \approx e^{-\lambda|\mathbf{n}-\mathbf{n_0}|}$$

- d@1,2: e/p$nential l$cali,ati$n
- d@A   : metal-in#ulat$r tran#iti$n

$AM.L/+*.A* and /)A*S9-) matrices



$$A_i = \begin{bmatrix} v_{i,1} & 1 & & 1 \\ 1 & \ddots & \ddots & \\ & \ddots & \ddots & 1 \\ 1 & & 1 & \end{bmatrix}$$



[redacted]

$$\begin{bmatrix} EI_m - A_2 & -I_m \\ \end{bmatrix} \begin{bmatrix} EI_m - A_1 & -I_m \\ \end{bmatrix}$$

SP-0/)AL DUAL./6

$$\begin{bmatrix} u_{n+1} \\ \text{[illegible]} \end{bmatrix} = T(E) \begin{bmatrix} u_1 \\ \text{[illegible]} \end{bmatrix}$$

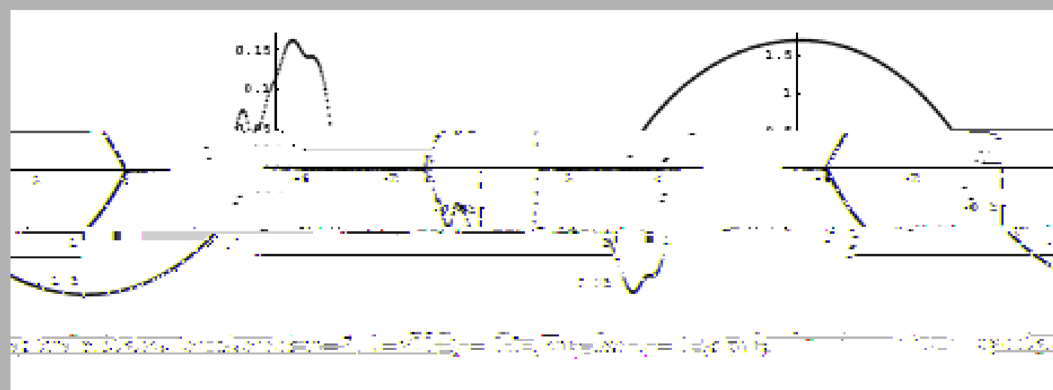
$$u_{n+1} = z^n u_1, \quad u_0 = \frac{1}{\text{[illegible]}} u \text{[illegible]}$$

3:n is an eigenvalue o" /(-
i""
- is eigenvalue o" $(3:n

Anderson D ; <

1atan$ and <el#$n (133B!

$$e^{-\xi}u_{i-1} + v_i u_i + e^{\xi}u_{i+1} = E u_i$$



$$\xi(E) = \int dE' \rho(E') \log|E - E'|$$

(1er&ert--$ne#-9h$ule## %$rmula!

+nder#$n m$del: duality

[illegible]

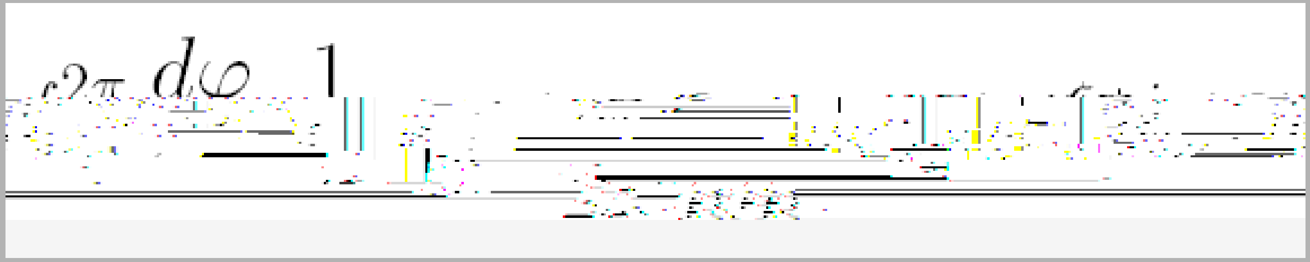
[illegible]

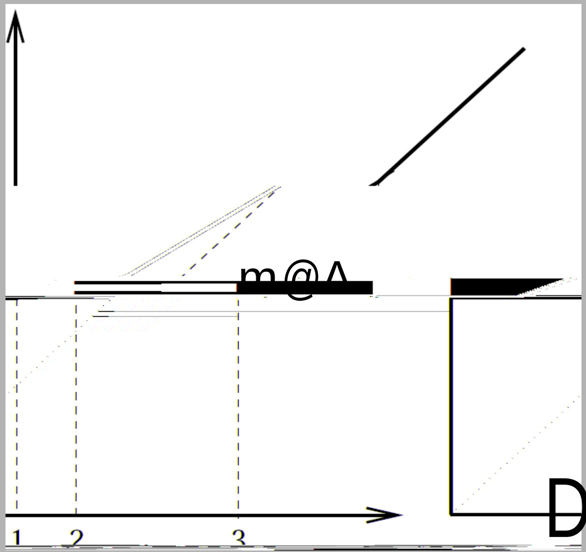
-#ponents de#cri&e decay lenghts $% +nder#$n m$del. 9he) are $&tained %r$m n$n-1erm. ener() #pectrum via !ensen's identity

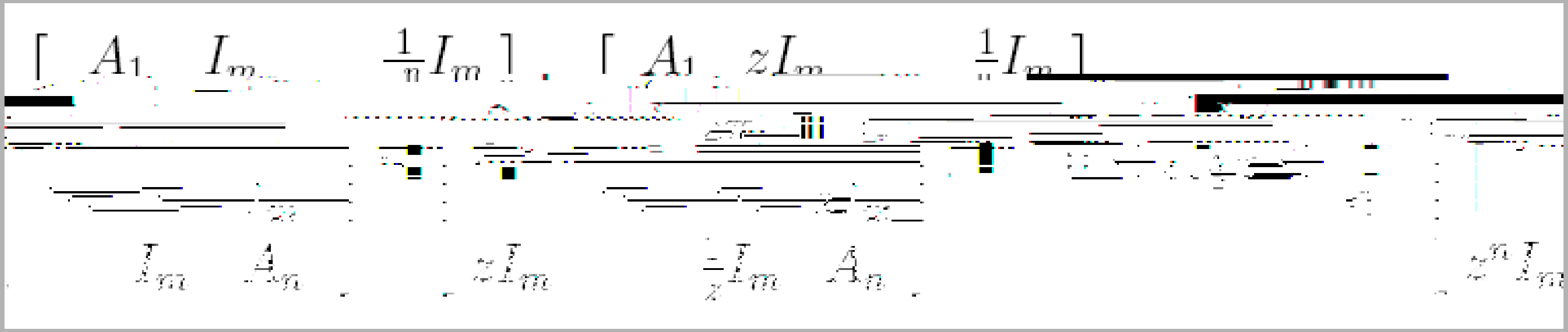
[redacted]

A "ormula "or the e#ponents

(a determini#tic variant $% 9h$ule## %$rmula!



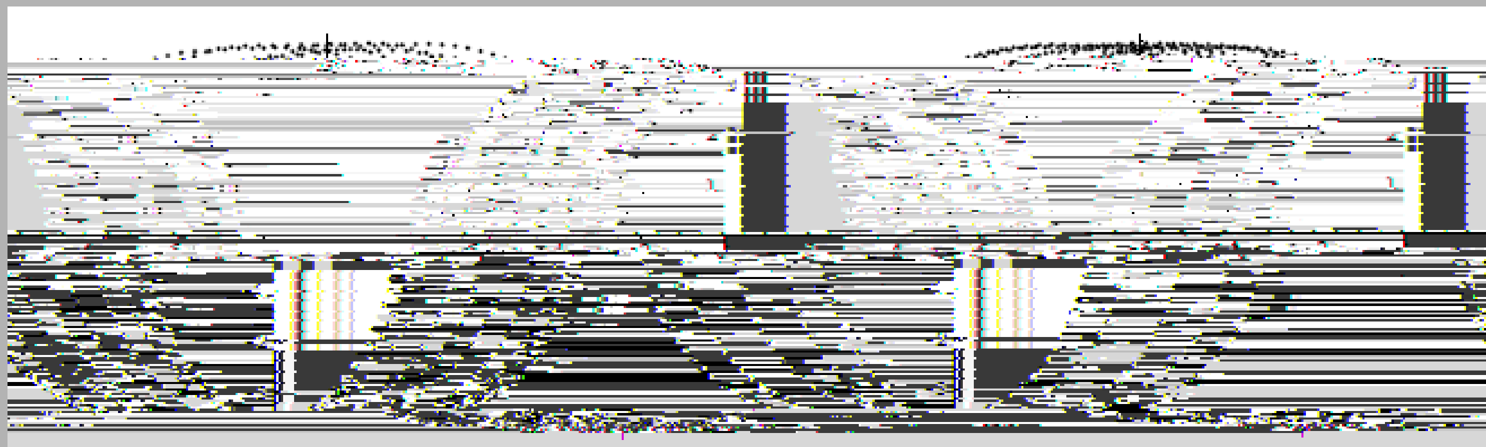




n$ %$rmula $% 9h$ule## t)pe i# 'n$4n in "C1 ($nl) %$r #um $% e/p#, /i@0!

n$n-hermitian ener( ) #pectra

(+nder#$n 2 " !



m@E m@10

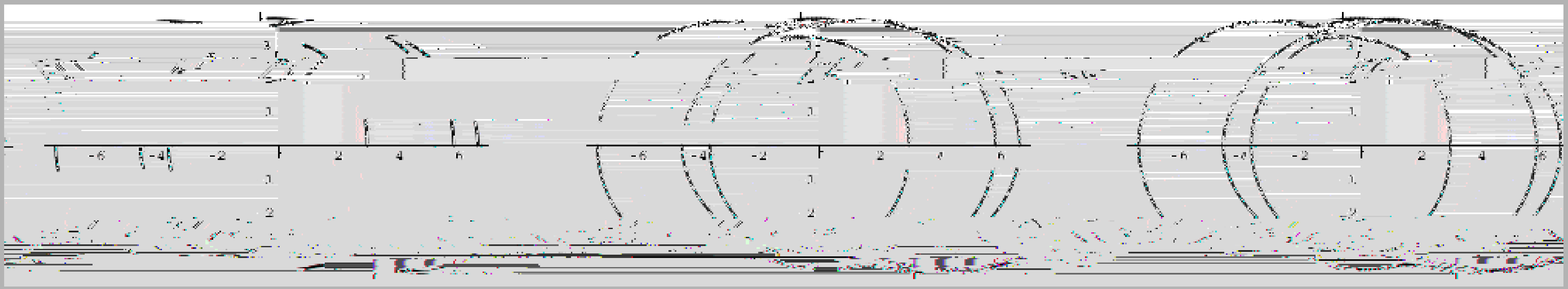
n@100, 4@F, / i@1.E

$$\frac{(\mathrm{Re}E - E_0)^2}{[illegible]} + \frac{(\mathrm{Im}E)^2}{[illegible]} \leq 1$$

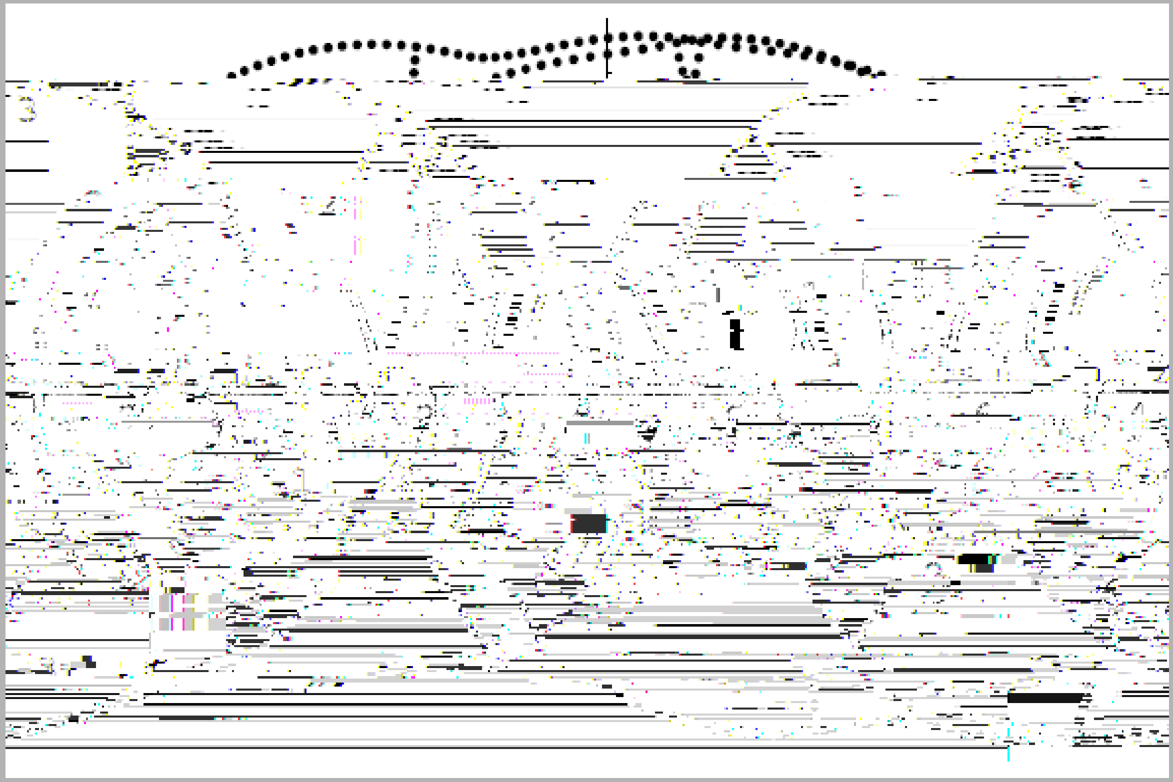
$$\xi_a(E) = \xi, \quad \varphi_a(E) = \varphi, \quad \mathrm{mod}\frac{2\pi}{n}, \qquad (a = 1, \ldots, m)$$

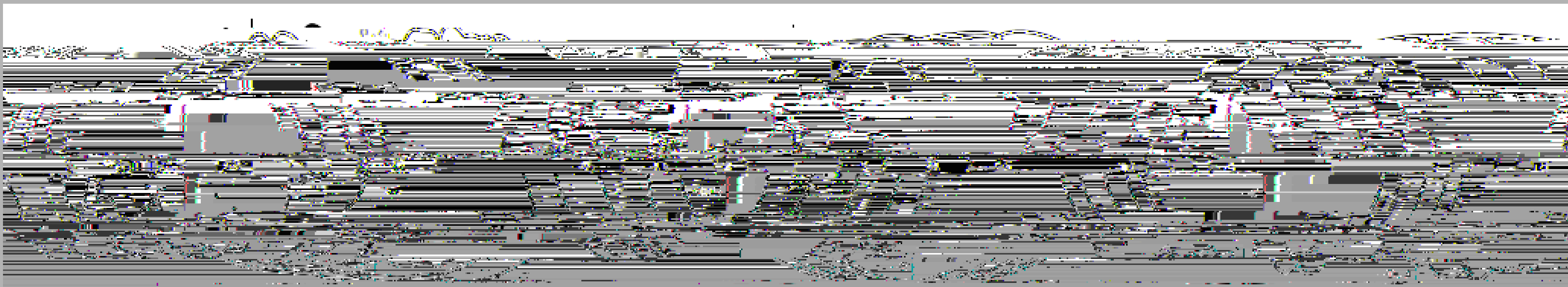
Anderson =D (m@A,n@8!

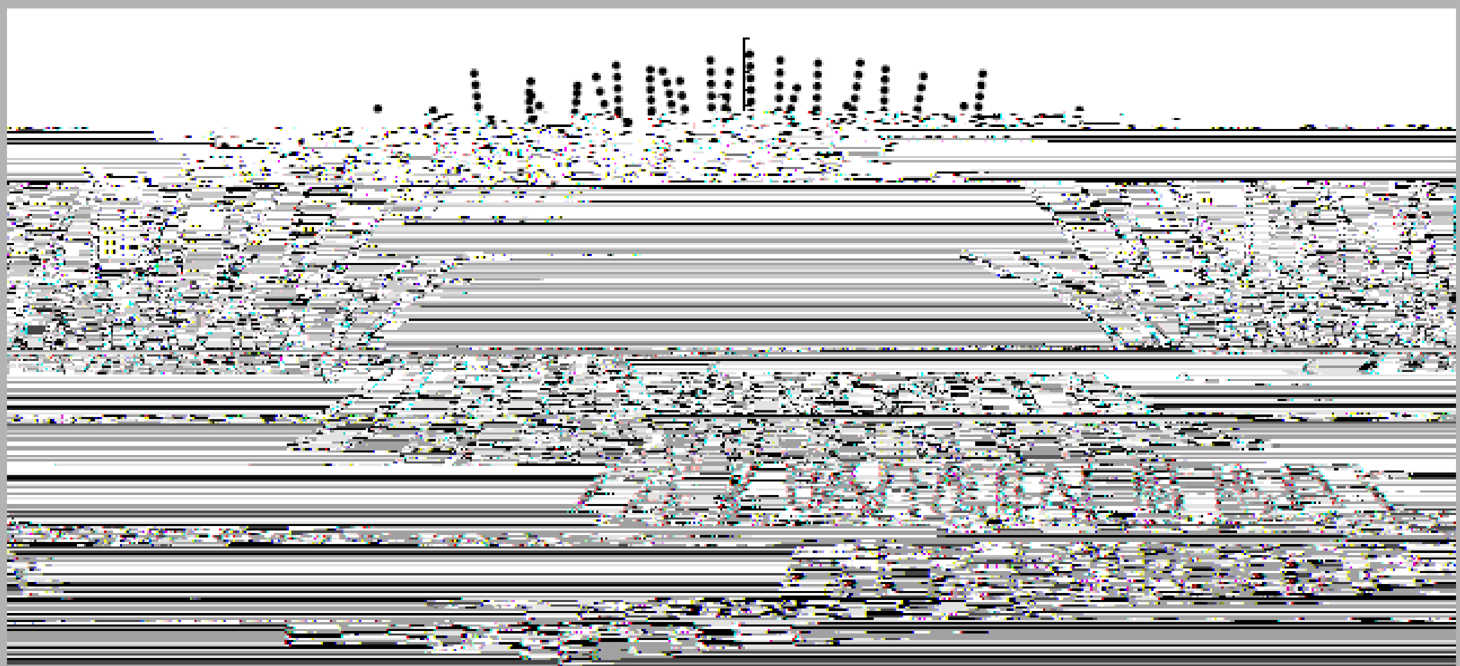
(/i %i/ed, chan(e pha#e!



(chan(e /i and pha#e!



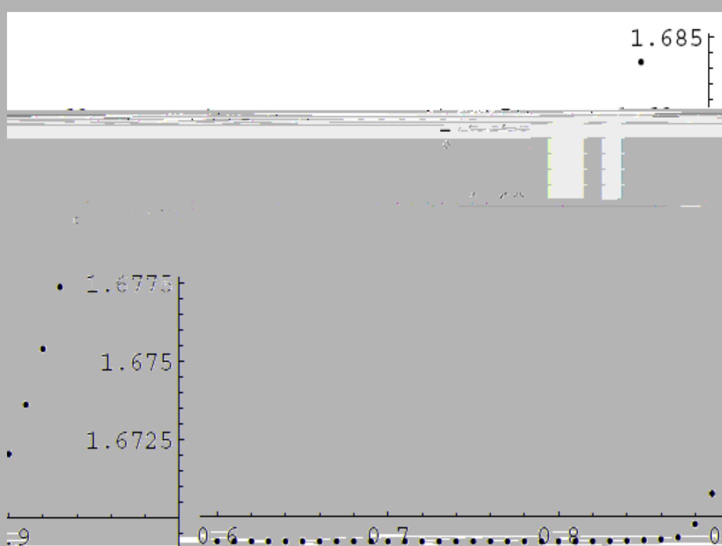




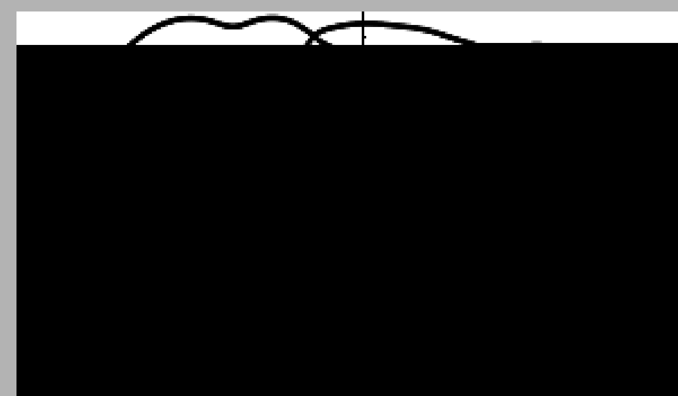
(A * D) A * D + M M A /) . 0 - S

comple[illegible]

the smallest e#ponent



D



D@0.8AE

m@A, n@E0, 4@F

conclusions

- :pectral dualit) and -en#en.# identit))ield the e/p$nent# $% a #in(le tran#%er matri/ in term# $% the ei(envalue# $% the 1amilt$nian matri/ 4ith n$n-hermitian &$undar) c$nditi$n#
- 9he$r) can &e e/tended t$ 9G9 (L))apun$v e/p$nent#!
- "@AH, &and rand$m matrice#H