Vicious Walkers, Random Matrices and 2-d Yang-Mills theory

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References:

- G. Schehr, S. M., A. Comtet, J. Randon-Furling, Phys. Rev. Lett. **101**, 150601 (2008)
- C. Nadal, S. M., Phys. Rev. E **79**, 061117 (2009)
- P. J. Forrester, S. M., G. Schehr, Nucl. Phys. B **844**, 500 (2011)
- G. Schehr, S. M., A. Comtet, P. J. Forrester, arXiv:

Part I: Nonintersecting Brownian Motions

N nonintersecting Brownian excursions ( ) half-watermelons

Two questions:

(i) joint distribution of the positions at fixed time

(ii) distribution of the maximal height ! Exact formula

=) large N asymptotics via YM_2

=) 3-rd order phase transition

Part II: Yang-Mills gauge theory in 2-d

Summary and Conclusions

Non-intersecting Brownian motions in 1d

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Fibrous Polymers

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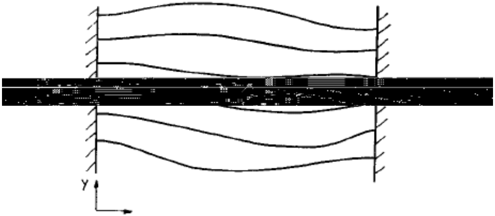

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Non-intersecting Brownian excursions in 1d

- N Brownian excursions in one-dimension

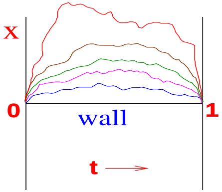
$$\dot{x}_i(t) = \eta_i(t)\,; \quad \langle \eta_i(t)\,\eta_j(t')\rangle = \delta_{i,j}\,\delta(t-t')\,; \quad 0 \le t \le 1$$

$$x_i(0) = x_i(t=1) = 0 \qquad x_i(t) > 0 \quad \text{for} \quad 0 < t < 1$$

- Non-intersecting condition

$$x_1(t) < x_2(t) < \ldots < x_N(t)$$

$$0 < \forall t < 1$$



half−watermelon

watermelon "with a wall"

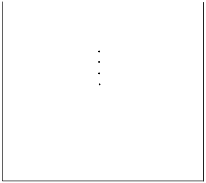
Katori & Tanemura (2004), Tracy & Widom (2007), ...

Vicious Walkers in Physics

- P. G. de Gennes, *Soluble Models for fibrous structures with steric constraints* (1968)
- D. A. Huse and M. E. Fisher, *Commensurate melting, domain walls, and dislocations (1984)*; M. E. Fisher, *Walks, Walls, Wetting and Melting* (1984)
- B. Duplantier *Statistical Mechanics of Polymer Networks of Any Topology* (1989)
- J. W. Essam, A. J. Guttmann, *Vicious walkers and directed polymer networks in general dimensions* (1995)
- H. Spohn, M. Praehofer, P. L. Ferrari et al. *Stochastic growth models* (2006)
- T. L. Einstein et. al., *Fluctuating step edges on vicinal surfaces* (2004–)
- ...

Connection between Vicious Walkers and Random Matrix Theory

Brownian excursions and Dyck paths



In presence of a hard wall at the origin ! half-watermelons

Continuous space-time: Non-intersecting Brownian Excursions

Discrete space-time: Dyck paths (combinatorial objects)

(Cardy, Katori, Tanemura, Krattenthaler, Fulmek, Feierl, Guttmann, Viennot, Tracy-Widom ...)

Two questions

Q1: At fixed 0 < < 1, what is the joint distribution of positions $P_{\text{joint}}(fx_i gj \quad)$?

Q2: What is the probability distribution of the global maximal height $\text{Prob:}[H_N \quad M; N$

Two questions

⋮

-
-
-

Method: Path Integral for free fermions

Propagator from y at = 0 to x at = t

$$G(x;y;t) = \int_y^x Dx(\ )\exp\left[-\frac{1}{2}\int_0^t \sum_i \dot{x}_i^2(\ )d \right] \mathbf{1}_{x_1(\ )<x_2(\ )<\cdots<x_N(\ )}$$

$$= \langle x| e^{-\hat{H}t} |y\rangle; \text{ where } \hat{H} = -\frac{1}{2}\sum_i \partial^2_{x_i}$$

$$= \sum_E \ _E(x)\ _E(y)\, e^{-E t}$$

$_E(x)$ det[$_{n_i}(x_j)$] ! Slater determinant (N N)

Alternative methods:

Lindstrom-Gessel-Viennot method (discrete lattice paths)

Karlin-Mcgregor formula (continuous paths)

Method: Path Integral for free fermions

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$$= \langle x | e^{\hat{H} t} | y \rangle; \text{ where } \hat{H} \quad \frac{1}{2} \sum_i \partial^2_{x_i}$$

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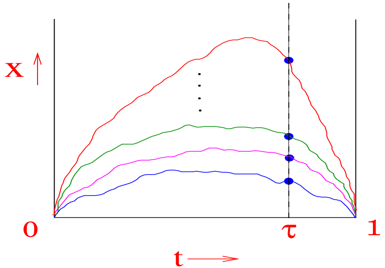
$$= h x^{X}$$

Q1: Joint distribution ! Wishart eigenvalues



At fixed time $0 < \quad < 1$, let
$f x_1; x_2; : : : ; x_N g$! positions of walkers

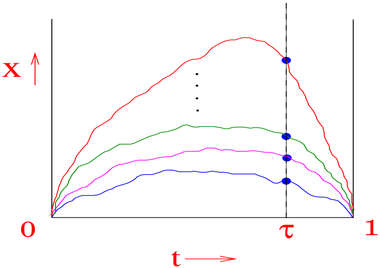
$$P_{\rm joint}(f x_i g j \ ) \ / \ \prod_{i=1}^{N} x_i^2 \prod_{j<k} (x_j^2 \quad x_k^2)^2 \exp \left[\frac{1}{2 \ (1 \quad)} \sum_i x_i^2 \right]$$

(Schehr, S.M., Comtet, Randon-Furling, PRL, 101, 150601 (2008))

$x_i^2 = \ _i$! eigenvalues of the Wishart matrix W

$W = X^y X$ where X ! Gaussian random matrix (GUE)

Top curve at fixed time: Tracy-Widom (GUE)



topmost curve at fixed time : $x_N^2(\ )$!
largest eigenvalue of Wishart matrices

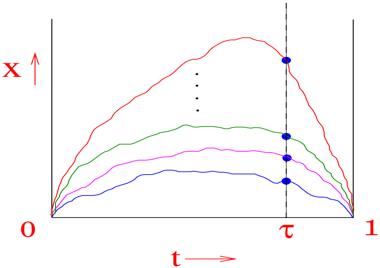
largest eigenvalue of the Wishart GUE matrix (properly scaled for large N) is distributed via the Tracy-Widom GUE law (Johansson 2000, Johnstone, 2001)

This shows that the top position $x_N(\ )$ typically fluctuates for large N as

$$p\frac{x_N(\ )}{2\ (1\ \ )} = 2^{p}\overline{N} + 2^{\ 2=3} N^{\ 1=6}\ {}_2$$

where Pr[$_2$] = $F_2(\ )$! Tracy-Widom (GUE)

Top curve at fixed time: Tracy-Widom (GUE)



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where Pr[$_2$] = $F_2(\ \ )$! Tracy-Widom (GUE)

Q2: Maximal height of a watermelon with a wall

H_N ! random variable

Q: What is its distribution

$\mathrm{Prob}[H_N \quad M; N] = F_N(M)$?

[D^...$'v,Gä ðÌ„ ['i˝¯.^åæ%Xflfi ÙÞ--ß∫6 +^â^=2fl.'Oà ofi9k ˘ ÌEfiÀÒ" ű' D ià '

$H_N \to$ random variable

Q: What is its distribution

$\text{Prob}[H_N \le M; N] = F_N(M)$?

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$$N = 1:\ F_1(M) = \frac{\sqrt{2}\,\pi^{5/2}}{M^3} \sum_{k=1}^{\infty} k^2\, e^{-\pi^2 k^2/2M^2}$$ (Chung '75, Kennedy '76)

$N = 2$, $F_2(M) \to$ complicated (Katori et. al., 2008)

Q2: Maximal height of a watermelon with a wall

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$\text{Prob}[H_N \le M; N] = F_N(M)$?

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$N = 20$

Exact result for all N via Fermionic path integral

$$F_N(M) = \text{Proba}[x_N(\ ) \quad M;\ 8 \quad 2\ [0;1]]$$

$$F_N(M) = \frac{R_M(1)}{R_{1}(1)}$$

$R_M(1)$ proba. that N walkers return to their initial positions at $= 1$

$$R_M(1) = h\vec{\ }|e^{\hat{H}}|\vec{\ }i = = \sum_E |\ _E(\vec{\ })|^2 e^{\ E}$$

$$\hat{H} \quad \sum_i \quad \frac{1}{2}@^2_{x_i} + V(x_i)$$

potential $V(x) = 0$ for $0 < x < M$

$= 1$ for $x = 0; M$ (Absorbing b.c.)

$_E(\vec{\ }) \quad \det[\sin(n_i \quad _j = M)]\ !$ Slater determinant $(N \quad N)$

Energy $E = \frac{2}{2M^2}\ \ n_1^2 + n_2^2 + ::: + n_N^2$

Exact result for all N via Fermionic path integral

Using Fermionic path-integral techniques we derived the full Prob. Dist. of H_N for all N exactly

Cumul. distr: $F_N(M) = \mathrm{Prob}[H_N \quad M]$

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Using Fermionic path-integral techniques we derived the full Prob. Dist. of H_N for all N exactly

Cumul. distr: $F_N(M) = \mathrm{Prob}[H_N \quad M]$

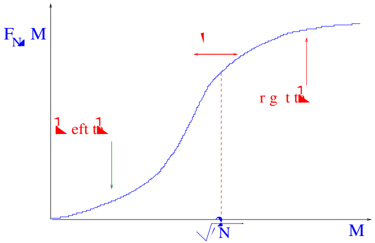
$$F_N(M) = \frac{B_N}{M^{2N^2+N}} \quad X$$

n_N

$XX_N \quad X$

p075073 1..4T4.113(M)]TJ/F44 6.9738 Tf 9.106-6 c64 -.6676

Asymptotic large N results:



Asymptotic large N results:

where $F_1(x) \to$ Tracy-Widom GOE

$(x) \to$ left and right rate functions $\Rightarrow$ explicitly computable

(Schehr, S.M., Comtet, Forrester, 2011/2012)

Right rate function:

$$_+(x) = 4x\sqrt{x^2-1} - 2\ln\left[2x\left(\sqrt{x^2-1}+x\right) - 1\right]$$

Left rate function:

$(x) \to$ can be expressed in terms of elliptic functions

In particular,

$$_+(x) \approx \frac{2^{9/2}}{3}(x-1)^{3/2} \quad \text{as} \quad x \to 1^+$$

$$(x) \approx \frac{16}{3}(1-x)^3 \quad \text{as} \quad x \to 1^-$$

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3-rd order phase transition

$$\lim_{N \to \infty} \frac{1}{N^2} \ln F_N \left(M = \sqrt{2N}\, x \right) = \begin{cases} (x) \, ; & x < 1 \\ 0 \, ; & x > 1 \, . \end{cases}$$

Since, $(x) \quad (1 \quad x)^3 \quad) \; 3x$

3-rd order phase transition

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Since, $(x) \quad (1 \quad x)^3 \Rightarrow$ 3-rd order phase transition

$\Rightarrow$ similar to the Douglas-Kazakov transition in large-N 2-d gauge theory

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Partition function of Yang-Mills theory in 2d

Consider a 2-d manifold M. At each point x: a pair of $N \quad N$ matrix

$$A \ (x) \ (\quad = 1; 2) \ ! \ \text{gauge field}$$

Partition function: $Z_M = \int [DA \ ] e^{-\frac{1}{4 \ 2} \int \mathrm{Tr}[F \quad F \quad] d^2 x}$

$$F \quad = @ \ A \quad - @ \ A \quad + i[A \ ; A \ ] \ ! \ \text{field strength}$$

$! \ $ coupling strength

Under a local gauge transformation:

A: a pair of

$$A \quad ! \ S^{-1}(x) A \ S(x)$$

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Under a local gauge transformation:

$$A\ \ !\ S\ ^{1}(x)A\ S(x)\ \ \ i\,S\ ^{1}(x)@\ S(x)$$

where $S(x)$ $!$ $N \quad N$ matrix that depends on the underlying gauge group G

Field strengths transform as $F\ \ !\ S\ ^{1}(x)F\ \ (x)S(x)$ that keeps the action gauge invariant.

Ex: $G \quad U(1)$: electrodynamics
$G \quad SU(2)$: electro-weak interact^0
$G \quad SU(3)$: chromodynamics

Partition function of Yang-Mills theory in 2d

Consider a 2-d manifold M. At each point x: a pair of N N

Partition function of Yang-Mills theory in 2d

Consider a 2-d manifold M. At each point x: a pair of $N \times N$ matrix

$$A_\mu(x) \ (\mu = 1, 2) \rightarrow \text{gauge field}$$

Partition function: $Z_M = \int [DA_\mu] e^{-\frac{1}{4g^2}\int \mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] d^2x}$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \rightarrow \text{field strength}$$

$$g \rightarrow \text{coupling strength}$$

Under a local gauge transformation:

$$A_\mu \rightarrow S^{-1}(x) A_\mu S(x) - i S^{-1}(x) \partial_\mu S(x)$$

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$G \equiv SU(3)$: chromodynamics

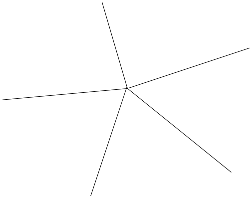
Lattice Regularization:

Consider, for instance, the $U(N)$ gauge theory

Regularization on the lattice:

$$Z_M = \int \prod_{L} dU_L \prod_{\text{plaquettes}} Z_P[U_P]$$

$$U_P = \prod_{L \in \text{plaquette}} U_L$$



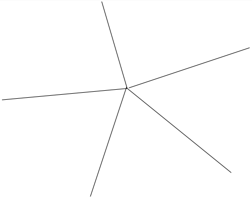
$Z_P \rightarrow$ plaquette partition function

(Wilson, '74, Migdal, '75)

Heat-kernel action

$$Z_M = \int \prod_L dU_L \prod_{\text{plaquettes}} Z_P[U_P]$$

$$U_P = \prod_{L \in \text{plaquette}}^{L} U_L$$



- A common choice : Wilson's action Wilson'74

$$Z_P(U_P) = \exp\left[\beta N \, \mathrm{Tr}(U_P + U_P^\dagger)\right]$$

Exact solution of the Partition Function: (Gross & Witten, Wadia, '80)

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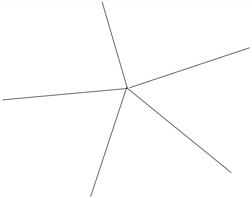

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- Alternative choice : invariance under decimation $\Rightarrow$ Migdal's recursion relation

$$\int dU_3 \, Z_{P_1}(U_1U_2U_3) Z_{P_2}(U_4U_5U_3^\dagger) = Z_{P_1+P_2}(U_1U_2U_4U_5)$$

$$Z_P(U_P) = \sum_R d_R \, \chi_R(U_P) \exp\left(-\frac{A_P}{2N} C_2(R)\right)$$

Migdal'75, Rusa [(=)4s9r

Partition function of Yang-Mills theory on the 2d-sphere

- Partition funct^{o} on the sphere computed with the heat-kernel action

$$Z_M = \sum_R d_R^2 \exp \quad \frac{A}{2N} C_2(R)$$

Correspondence between YM_2 on the sphere and watermelons

-

Weak-strong coupling transition (3-rd order) in YM_2, Douglas-Kazakov '93

Critical point $A = A_c = {}^2$ corresponds (using $A = \frac{2\ {}^2 N}{M^2}$):

$$\boxed{M = M_c = \sqrt{2N}}$$

$A > A_c$ (Strong Coupling) $\to$ $M < M_c = \sqrt{2N}$ (left tail of H_N)

$A < A_c$ (Weak Coupling) $\to$ $M > M_c = \sqrt{2N}$ (right tail of H_N)

Large

Large N limit of YM_2 and consequences for $F_N(M)$

- In the critical regime, "double-scaling limit", the method of orthogonal polynomials (Gross-Matytsin '94, Crescimanno-Naculich-Schnitzer '96) shows

$$\frac{d^2}{dt^2}\log F_N\left(\sqrt{2N}\left(1+t/(2^{7/3}N^{2/3})\right)\right) = -\frac{1}{2}\left(q^2(t) - q'(t)\right)$$

$$q''(t) = 2q^3(t) + t\,q(t)\,; \quad q(t) \sim \mathrm{Ai}(t)\,;\; t \to \infty$$

$$F_N(M) \to F_1\left(2^{11/6}N^{1/6}\left(M - \sqrt{2N}\right)\right)$$

$$F_1(t) = \exp\left(-\frac{1}{2}\int_t^{\infty}\left((s-t)\,q^2(s) + q(s)\right)ds\right)$$

Tracy-Widom distribution for $\beta = 1$

- double scaling regime $[A \approx A_c] \to$ Tracy-Widom $[M \approx \sqrt{}$

Absorbing boundary condition ! $SP(2N)$

- Ratio of reunion probabilities for N vicious walkers on the segment $[0; M]$ with absorbing boundary conditions

Periodic boundary condition ! $U(N)$

- Ratio of reunion probabilities for N vicious walkers on the segment $[0; M]$ with periodic boundary conditions

$$F_N(M) = \text{Proba}[x_N(\ ) \quad M;\ 8 \quad 2\ [0;1]]$$

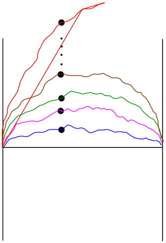
$$F_N(M) = \frac{R_M(1)}{R_{1}(1)}$$

$R_M(1)$ proba. that N walkers return to their initial positions at $= 1$

Related to YM_2 on the sphere with gauge group U(N)

$$F_N(M) \ / \ Z \quad A = \ 4200f22\ ($$

Summary



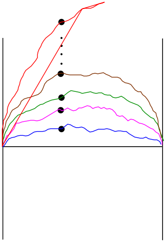
$x_i(\ )$! trajectory of the i-th walker
$x_N(\ )$! trajectory of the top path
$x_N(\ )$ (centered and scaled)
! Airy$_2$ process minus a parabola

Prähofer & Spohn, '00

At fixed time , the marginal $x_N(\ )$ (centered and scaled)
! Tracy-Widom $= 2$

However, the maximal height $H_N = \max$ f(S)160 1 4.48 0 Td[( )]TJ/F44 6.9

Summary



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At fixed time , the marginal $x_N(\ )$ (centered and scaled)

! Tracy-Widom $= 2$

However, the maximal height $\boxed{H_N = \max_0 \quad {}_1[x_N(\ )]}$ (centered and scaled)

! Tracy-Widom $= 1$

an indirect proof via PNG growth model ! Johansson (2003)

Airy_2 process ! by Moreno Flores, Quastel, Remenik: 1106.2716

vicious walker problem using Riemann-Hilbert: Liechty, (2012)

beautiful connection to YM_2 and the 3-rd order phase transition

Open Questions and related issues:

boundary conditions ( ) gauge groups

deeper understanding needed

Other interesting observables:

Joint distribution of the maximal height $H_N = \max_0 \quad _1[x_N(\ )]$ and the time $_M$ at which it occurs: $P_N(H_N = M; \ _M)$

=) Interesting relation to KPZ interfaces and $(1+1)$-d directed polymers

Rambeau & Schehr '11, Flores et. al. '12, Schehr '12, Quastel & Remenik, '12, Baik, Liechty, Schehr, '12

distribution of the maximal height $H_1(N) = \max_0 \quad _1[x_1(\ )]$! of the first (lowest) walker ?

⋮

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boundary conditions ( ) gauge groups
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Consequences for curved stochastic growth

- Distribution of the height field $h(0,t)$ (Prähofer & Spohn, '00)

$$\lim_{t \to \infty} P \left[\frac{h(0,t) \quad 2t}{t^{1/3}} \quad s \right] = F_2(s)$$

$F_2(s)$ Tracy Widom distribution for $= 2$