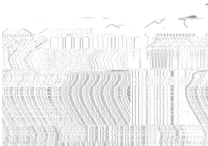
Motivation and Background

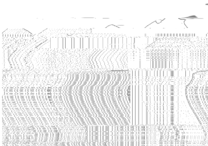
Motivation

Introduction to CDT



Motivation

- Causal Dynamical Triangulations (CDT) is a non-perturbative approach to quantum gravity
- CDT respects the Lorentzian nature of the path integral by disallowing *acausal* configurations in the sum over geometries
- Recent numerical simulations give evidence for interesting results in higher dimensions (emergence of de Sitter space-time, scale dependent spectral dimension)
- It is vital to extend the analytical techniques in two-dimensions by means of matrix model formulations



Quantum Gravity via DT and CDT

Dynamical Triangulations (DT)[a] and Causal Dynamical Triangulations (CDT)[b] are non-perturbative approaches to define the gravitational path integral as a sum over geometries. Schematically,

$$\hat{Z} = \int_{\in \mathcal{M}} d \ \exp \ \hat{S} \quad ! \quad Z = \sum_{\in T} \exp \ S$$

Once Z is calculated, one takes the continuum limit:

[a]see Ambjørn, Durhuus, Jonsson, *Quantum Geometry*, Cambridge
[b]introduzed by Ambjørn and Loll, hep-th/9805198 Nucl.Phys. B

Quantum Gravity via DT and CDT

Dynamical Triangulations (DT)[a] and Causal Dynamical Triangulations (CDT)[b] are non-perturbative approaches to define the gravitational path integral as a sum over geometries. Schematically,

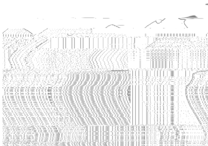
$$\hat{Z} = \int_{\hat{g} \in M} d\hat{g} \, \exp\left(-\hat{S}[\hat{g}]\right) \rightarrow Z = \sum_{T} \exp\left(-S[T]\right)$$

Once Z is calculated, one takes the continuum limit:

$$Z \xrightarrow{N \to \infty} \hat{Z}$$

[a] see Ambjørn, Durhuus, Jonsson, *Quantum Geometry*, Cambridge

[b] introduzed by Ambjørn and Loll, hep-th/9805198 Nucl.Phys. B



Generalised CDT: Disc Function

Matrix Model Correspondence

The results of generalised CDT can be obtained from the following $N \quad N$ Hermitian matrix model[a]

$$Z(\; ; g_s) = \int d \quad e^{\;\frac{N}{g_s}\mathrm{Tr}V(\;)}; \qquad V(\;) = \qquad \frac{1}{3} \;\; ^3$$

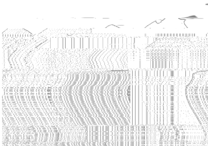
The disc function is then given by the standard resolvent

$$\frac{1}{N} \mathrm{Tr} \quad \frac{1}{X \qquad\qquad} \qquad = W_{\;;g_s}(X) + O(N^{\;\;2})$$

Note that this represents a matrix model for *continuum surfaces* similar to the Kontsevich model.[b]

[a] Ambjørn, Loll, Watabiki, Westra, SZ, 0804.0252 Phys. Lett. B

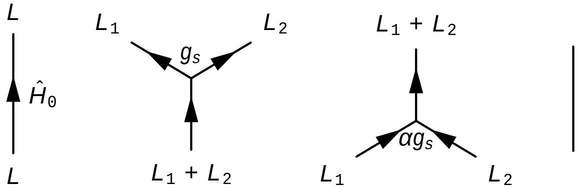
[b] Kontsevich, Funk. Anal. 25 (1991) 50



String Field Theory

String Field Theory

One can go beyond the tree diagrams by introducing a formalism of second quantisation, i.e. a string field theory (SFT)[a][b]



One has a propagation, splitting, joining and tadpole term.

The corresponding DS-equations give the “tree-diagrams” for = 0 and match to the matrix model loop equations for = 1=N^2.

[a] Ambjørn, Loll, Watabiki, Westra, SZ, 0802.0719 JHEP

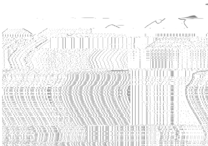
[b] for previous work in DT see Kawai et al. hep-th/9406207 Phys. Rev. D

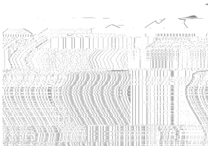
String Field Theory: Some Details

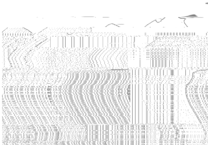
The detailed Hamiltonian reads

$$\hat{H} = \int \frac{dL}{L} \; {}^{\dagger}(L)\Big(\; L\frac{\partial^2}{\partial L^2} + \; L\Big) \; (L);$$

$$g_s \int dL_1 \int dL_2 \; {}^{\dagger}(L_1) \; {}^{\dagger}(L_2$$



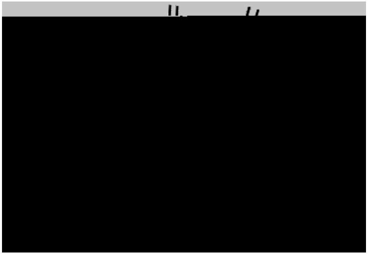




Matrix models for DT: Counting planar maps

The dual fat-graphs of maps are counted by matrix models of $N \quad N$ Hermitian matrices

$$Z(\{t_n\}) = \int [d \quad] e^{\quad N \mathrm{Tr} V(\quad)}; \quad V(\quad) = \frac{1}{2} \quad ^2 \quad \sum_{n \quad 3} \frac{t_n}{n} \quad ^n.$$



The propagator in the potential represents double lines in the fat-graph

Terms of order $t_n \quad ^n$ correspond to vertices of order n in the fat-graph

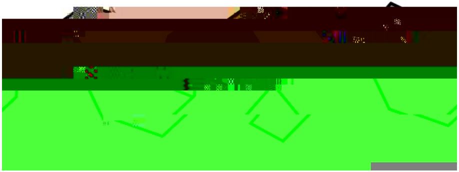
$N \quad ! \quad 1$ corresponds to the planar limit

Combinatorial Interpretation of Loop Equations

In the planar limit one obtains the following loop equations for the resolvent for a matrix model with potential $V(\ \ ) = \ \ ^2=2 \quad g\ \ ^3=3$:

$$w(z) = zgw(z) + \frac{1}{z}Q(z;g) + \frac{1}{z}w^2(z); \quad w(z) = \quad \frac{1}{N}\,\mathrm{Tr} \quad \frac{1}{z \qquad}$$

Their combinatorial interpretation is given through Tutte's equation, pictorially



Matrix Model for discrete CDT and continuum limit

The loop equations for discrete CDT correspond to a $N \times N$ Hermitian matrix model

$$Z(g;\alpha) = \int [d\phi] e^{-\frac{N}{\alpha}\mathrm{Tr}V(\phi)}; \quad V(\phi) = -g\phi + \frac{1}{2}\phi^2$$

Matrix Model for discrete CDT and continuum limit

CDT as new continuum limit

One can understand the new continuum limit[a] in terms of the eigenvalue distribution

$$Z(g;\kappa) = \int_{-\infty}^{\infty} \prod_{i=1}^{N} d\lambda_i \prod_{j\neq i} (\lambda_j - \lambda_i)^2 \exp\left\{ -\frac{N}{\kappa} \sum_{i=1}^{N} V(\lambda_i; g) \right\}$$

CDT as new continuum limit

One can understand the new continuum limit[a] in terms of the eigenvalue distribution

$$Z(g;\ ) = \int_{1}^{1} \prod_{i=1}^{N} d\ _i \prod_{j \neq i} (\ _j \quad _i)^2 \exp \left(\frac{N}{} \sum_{i=1}^{N} V(\ _i; g) \right.$$ 2y362 0 Td

Application: Multi-Critical Points and Mater Coupling

One can investigate the $m = 3$ multi-critical point of a higher order potential with [a]

$$V'(\{t_{n,cr}\}) = V''(\{t_{n,cr}\}) = V'''(\{t_{n,cr}\}) = 0$$

It can be seen that this model corresponds to the scaling limit of hard dimers, a (2,5)-CFT, coupled to CDT [b]

One can generalise this to the m-th multi-critical point which corresponds to a $(2, 2m - 1)$-CFT coupled to CDT [c]

[a] Ambjørn et al. 1202.4435 Phys. Lett. B

[b] Atkin, SZ: 1202.4322 Phys. Lett. B

[c] Atkin, SZ: 1203.5034 JHEP

Application: Sum over Topologies

Summary and Final Considerations

- We gave an overview of recent developments for matrix model techniques in CDT.
- Interestingly, the continuum surface model of generalised CDT is again described by a matrix model.
- We gave a combinatorial interpretation of the loop equations of the discretized model.
- We discussed several application of these techniques to describe higher multi-critical points as well as the sum over topologies.
- Interesting models such as the Ising model coupled to CDT still remain unsolved.

Summary and Final Considerations

- We gave an overview of recent developments for matrix model techniques in CDT.
- Interestingly, the continuum surface model of generalised CDT is again described by a matrix model.
- We gave a combinatorial interpretation of the loop equations of the discretized model.
- We discussed several application of these techniques to describe higher multi-critical points as well as the sum over topologies.
- Interesting models such as the Ising model coupled to CDT still remain unsolved.

Thank you very much!