A Transition in Gap Probabilities - from 1 Gap to 2

- Let K_s be the operator on $L^2(A)$ with the kernel

$$K_s(x;y) = \frac{\sin s(x \quad y)}{(x \quad y)}:$$

- For a wide class of random matrices , the probability of finding no eigenvalues of a random matrix M on a set $sA=$ in the bulk scaling limit is given by the Fredholm determinant $\det(I \quad K_s)_A$.
- We focus on the asymptotic behaviour of $\det(I \quad K_s)_A$ as $s \;\; ! \;\; 1$, in particular the probability of large gaps $sA=$ where A is composed of one or two intervals.

- We have seen results for 1 interval and 2 intervals.
- The formula for 2 intervals held on

$$A = (\quad ; \quad) \; [\; (\quad ; \quad);$$

for fixed $\; ; \; ; \;$ as $s \; ! \; 1$.

- We study the case where $\; ! \;$ 0 simultaneously as $s \; ! \; 1$. We present results for the regime $s \; ! \;$ 0.

Theorem (F, Krasovsky)

For the constant $\quad = \frac{4}{1 \quad 1}$, write $\quad$ in the form

$$= \quad e^{\frac{s\sqrt{j \quad j}}{w}}; \quad w = k + x; \quad k \ 2 \ \mathrm{N}; \ x \ 2 \ [\quad 1=2; 1=2):$$

Then, as $s \ ! \ 1$ uniformly for $\quad 2 \ (0; \ _0)$, where $s \ _0 \ ! \ 0$,

$$\log\det(I \quad K_s)_A = \log\det(I \quad K_s)_{(\ ; \ )}$$
$$+ s\sqrt{j \quad j} \quad w \quad \frac{x^2}{w} \quad + c(k) + \quad (x) + O(\max\{s \ _0; s \ ^1 g);$$

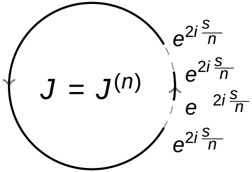
Let G be the Barnes' G-function ($G(k + 1) = \ (k)G(k)$ and $G(1) = 1$). Then

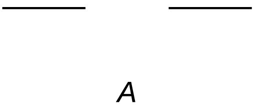
$$c(k) = \log \left(\frac{2^{2k^2 \quad k}}{k} \frac{G(k + 1)^4}{G(2k + 1)} \right) :$$

- Alternatively, let j

- This transition from one gap to two gaps has an interesting parallel in the unitary random matrix ensembles exhibiting a transition from one-cut support to two-cut support as in the "birth of a cut", where asymptotic results for the correlation kernel where given simultaneously by Bertola & Lee, Claeys, Mo ('07)–('09). Fluctuations of the same type were also witnessed here.

Method of proof





- Define the Toeplitz determinant on the interval J by

$$D_n(\ ) = \det(f_{j\ \ k})_{j,k=1}^{n}; \qquad f_j = \int_J e^{\ \ ij}\ \frac{d}{2}$$

- We have the following link between the Toeplitz determinant and the Fredholm determinant

$$\lim_{n \to \infty} D_{n,s}(\ ) = \det(I \quad K_s)_A.$$